Optimal spin-quantization axes for the polarization of dileptons with large transverse momentum

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The leading-order parton processes that produce a dilepton with large transverse momentum predict that the transverse polarization should increase with the transverse momentum for almost any choice of the quantization axis for the spin of the virtual photon. The rate of approach to complete transverse polarization depends on the choice of spin quantization axis. We propose axes that optimize that rate of approach. They are determined by the momentum of the dilepton and the direction of the jet that provides most of the balancing transverse momentum.

The spins of particles produced in high energy collisions carry important information about the fundamental interactions between elementary particles, but it is difficult to access that information. Since the spin cannot be measured directly, information about the spin of a particle must be inferred from the angular distribution of its decay products. The accessibility of information about the spin is affected by the choice of a spin quantization axis (SQA), because some angular variables are more easily measured than others. Hadron collisions provide an additional complication, because the fundamental interactions involve collisions of partons with varying longitudinal momenta. Integration over these longitudinal momenta tends to dilute the information carried by the spins of the final-state particles. This raises an important question: which SQA will maximize that information?

The simplest process for which this question can be addressed is the production of a *dilepton*, a lepton and antilepton with opposite charges. There are various sources of dileptons in high energy collisions, but we focus on production through a virtual photon, which can be regarded as a spin-1 particle with a variable mass equal to the invariant mass of the lepton pair. There are some simple dilepton production mechanisms that predict that the virtual photon should be transversely polarized. For the *Drell-Yan mechanism*, the annihilation of a quark and an antiquark into a virtual photon, the vector coupling to the quark and antiquark ensures that the virtual photon is transversely polarized [1]. The leading-order parton processes for a dilepton with large transverse momentum are dominated asymptotically by *photon fragmentation*, the production of a transversely-polarized real photon followed by its decay into a collinear lepton pair [2]. For these examples, the leading-order prediction of perturbative QCD is that the virtual photon is transversely polarized. Longitudinal virtual photons are produced by higher-order processes at a rate that depends on the SQA. A reasonable criterion for an optimal SQA is that it minimizes the cross section for producing a longitudinal virtual photon.

For the Drell-Yan mechanism, an optimal SQA was identified long ago: it is the *Collins-Soper axis* [3]. In this paper, we derive optimal SQA's for the parton processes that create a dilepton with large transverse momentum. We apply them to dilepton production at Fermilab's Tevatron and at CERN's Large Hadron Collider (LHC).

We consider the production of a dilepton whose invariant mass Q is well below the Z^0 resonance. The orientation of the lepton momentum in the dilepton rest frame is given by a polar angle θ with respect to a chosen axis and an azimuthal angle ϕ with respect to a chosen plane containing that axis. If ϕ is integrated over, the differential cross section can be expressed as

$$\frac{d\sigma}{dx} = \frac{\alpha}{4\pi} \int \frac{dQ^2}{Q^2} \left[\sigma_T \frac{1+x^2}{2} + \sigma_L (1-x^2) \right], \quad (1)$$

where $x = \cos\theta$ and σ_T and σ_L are the cross sections for a transverse and a longitudinal virtual photon, respectively. The polar axis that defines the angle θ can be identified with the SQA of the virtual photon. Our question can then be restated: for which SQA is the virtual photon most strongly polarized?

The longitudinal polarization 4-vector ϵ_L for a virtual photon with 4-momentum Q must satisfy $Q \cdot \epsilon_L = 0$ and $\epsilon_L^2 = -1$. The most general 4-vector satisfying these conditions can be written in the form

$$\epsilon_L^\mu = \tilde{X}^\mu / \sqrt{-\tilde{X}^2}, \quad \tilde{X}^\mu = (-g^{\mu\nu} + Q^\mu Q^\nu / Q^2) X_\nu, \quad (2)$$

where X is a 4-vector. The physical interpretation of X is that in the dilepton rest frame, which is defined by $\boldsymbol{Q} = 0$, the direction of the 3-vector $-\boldsymbol{X}$ is the SQA of the virtual photon.

We now consider the production of a dilepton in the collision of two hadrons with 4-momenta P_1 and P_2. The SQA is generally chosen to lie in the production plane defined by the momenta of the colliding hadrons in the dilepton rest frame. Thus X in Eq. (2) has the form

$$X^\mu = aP_1^\mu + bP_2^\mu, \quad (3)$$

where a and b are scalar functions. In the dilepton rest frame, the SQA is antiparallel to the unit vector $\hat{\boldsymbol{X}} = (a\boldsymbol{P}_1 + b\boldsymbol{P}_2)/|a\boldsymbol{P}_1 + b\boldsymbol{P}_2|$, so it is determined by the ratio a/b. If the leptons are the only particles in the final state whose momenta are measured, then a/b can only depend on the 4-vectors P_1, P_2, and Q. If additional information about the final state is measured, a/b can also depend on this information.

If the invariant mass $Q \equiv (Q^2)^{1/2}$ of the dilepton is large compared to the scale Λ_{QCD} of nonperturbative effects in QCD, the inclusive cross section can be calculated using QCD factorization formulas. Suppose the cross section is dominated by a specific parton process which, at leading order in the QCD coupling constant, predicts that the virtual photon is transversely polarized for any choice of the SQA. Then a natural prescription for an optimal SQA is that it minimizes the longitudinal cross section from that parton subprocess at leading order. Such an optimal SQA will depend on the longitudinal momentum fractions of the colliding partons, which cannot be directly measured.

We first consider the production of a dilepton with large invariant mass $Q \gg \Lambda_{\mathrm{QCD}}$. The Drell-Yan mechanism is the parton process $q\bar{q} \to \gamma^*$. If the momenta of the quark and antiquark are collinear to those of their parent hadrons, the virtual photon is transversely polarized for any choice of the SQA. A rigorous QCD calculation of the transverse momentum distribution for the dilepton requires the resummation of the effects of the emission of soft gluons from the colliding partons [4]. One of the most important qualitative effects of soft-gluon emission is that it gives transverse momenta to the colliding partons. A simple model for these effects is the parton model with intrinsic transverse momentum, in which the momentum distribution of a parton is the product of the parton distribution and a function of the 2-dimensional transverse momentum vector $\boldsymbol{k}_\perp$. Collins and Soper calculated the angular distribution of the dilepton in this model [3]. The cross section for $q\bar{q} \to \gamma_L^*$, where γ_L^* is a longitudinal virtual photon, is

$$\hat{\sigma}_L = \frac{8\pi^2 e_q^2 \alpha \langle k_\perp^2 \rangle (a^2 x_2^2 + b^2 x_1^2)}{3Q^2 (ax_2 - bx_1)^2} \delta(x_1 x_2 s - Q^2), \quad (4)$$

where x_1 and x_2 are the longitudinal momentum fractions of the colliding partons and e_q is the electric charge of the quark. We have used the expression for the longitudinal polarization vector in Eqs. (2) and (3). The cross section has been expanded to second order in $\boldsymbol{k}_{1\perp}$ and $\boldsymbol{k}_{2\perp}$ and averaged over them using $\langle k_{n\perp}^i \rangle = 0$ and $\langle k_{n\perp}^i k_{n\perp}^j \rangle = \frac{1}{2} \langle k_\perp^2 \rangle \delta_\perp^{ij}$, where $\delta_\perp^{ij}$ is the unit tensor in the transverse dimensions.

Collins and Soper proposed a convenient set of angles θ and ϕ for the lepton momentum in the dilepton rest frame [3]. Lam and Tung pointed out that the Collins-Soper axis is an optimal SQA for the Drell-Yan mechanism [5]. It minimizes the cross section for longitudinal

virtual photons in Eq. (4), thus making the leading-order prediction that the virtual photon will be transversely polarized as robust as possible with respect to radiative corrections that generate transverse momenta for the colliding partons. Minimizing the cross section in Eq. (4) with respect to the ratio a/b, we find

$$a/b\big|_{\mathrm{CS}} = -x_1/x_2. \quad (5)$$

Under the assumption that the cross section is dominated by the Drell-Yan mechanism $q\bar{q} \to \gamma^*$, we can derive an expression for x_1/x_2 in terms of variables that can be directly measured. At leading order in $\boldsymbol{k}_{n\perp}$, the energy-momentum conservation condition is $Q = x_1 P_1 + x_2 P_2$, which implies

$$x_1/x_2\big|_{\mathrm{CS}} = Q \cdot P_2 / Q \cdot P_1. \quad (6)$$

Equations (5) and (6) define the Collins-Soper axis. The corresponding 4-vector X in Eq. (3) becomes

$$X_{\mathrm{CS}}^\mu = \frac{P_1^\mu}{Q \cdot P_1} - \frac{P_2^\mu}{Q \cdot P_2}. \quad (7)$$

In the dilepton rest frame, the SQA is along $\boldsymbol{v}_1 - \boldsymbol{v}_2$, where the $\boldsymbol{v}_i$'s are the velocities of the colliding hadrons.

We now consider the production of a dilepton with large transverse momentum $Q_T \gg \Lambda_{\mathrm{QCD}}$. At leading order in α_s, the parton processes that create a virtual photon at large Q_T are $q\bar{q} \to \gamma^* g$, $qg \to \gamma^* q$, and $\bar{q}g \to \gamma^* \bar{q}$. For $Q_T \gg Q$, the cross section for a longitudinal virtual photon is suppressed by a factor of Q^2/Q_T^2. The coefficient of Q^2/Q_T^2 depends on the choice of SQA. Our prescription for the optimal SQA is to minimize the longitudinal cross section, which makes the transverse polarization for fixed Q^2 increase as rapidly as possible with increasing Q_T. We will refer to the optimal SQA for the parton process $ij \to \gamma^* k$ as the *optimal ij axis*.

We first determine the optimal $q\bar{q}$ axis. At leading order in α_s, the dependence of the differential cross section for $q\bar{q} \to \gamma_L^* g$ on a and b is

$$Q^0 \frac{d\hat{\sigma}_L}{d^3 Q} \propto \frac{a^2 x_2^2 + b^2 x_1^2}{(aw_1 + bw_2)^2 - abQ^2 s}, \quad (8)$$

where $w_1 = Q \cdot P_1$, $w_2 = Q \cdot P_2$, and s is the center-of-momentum (c.m.) energy of the colliding hadrons. There is a delta function constraint on these variables: $2(x_1 w_1 + x_2 w_2) = x_1 x_2 s + Q^2$. Minimizing Eq. (8) with respect to a/b, we get

$$\frac{a}{b}\bigg|_{q\bar{q} \to \gamma^* g} = \frac{x_1^2 w_1^2 - x_2^2 w_2^2 + Z^{1/2}}{x_2^2 (2w_1 w_2 - Q^2 s)}, \quad (9)$$
$$Z = (x_1^2 w_1^2 + x_2^2 w_2^2)^2 - x_1^2 x_2^2 Q^2 s (4w_1 w_2 - Q^2 s).$$

This also gives the optimal $\bar{q}q$ axis.

We next determine the optimal qg axis. At leading order in α_s, the dependence of the differential cross section

for $qg \to \gamma_L^* q$ on a and b is

$$Q^0 \frac{d\hat{\sigma}_L}{d^3Q} \propto \frac{(ax_2 - bx_1)^2 + b^2 x_1^2}{(aw_1 + bw_2)^2 - abQ^2 s}. \quad (10)$$

Minimizing with respect to a/b, we get

$$\left. \frac{a}{b} \right|_{qg \to \gamma^* q} = \frac{2x_1^2 w_1^2 - x_2^2 w_2^2 + Z^{1/2}}{x_2(2x_1 w_1^2 + 2x_2 w_1 w_2 - x_2 Q^2 s)}, \quad (11)$$
$$Z = (2x_1^2 w_1^2 + x_2^2 w_2^2 + 2x_1 x_2 w_1 w_2 - x_1 x_2 Q^2 s)^2$$
$$- x_1^2 x_2^2 Q^2 s(4w_1 w_2 - Q^2 s).$$

This also gives the optimal $\bar{q}g$ axis. The values of a/b for the optimal gq and $g\bar{q}$ axes are obtained by taking the reciprocal of the expression on the right side of Eq. (11) and then interchanging x_1 and w_1 with x_2 and w_2.

The expressions for a/b in Eqs. (9) and (11) depend on the longitudinal momentum fractions x_1 and x_2 of the colliding partons only through the ratio x_1/x_2. Our optimality criterion was based on the assumption that a specific parton process dominates. If that parton process implies a value for x_1/x_2 that depends on a measurable property of the final state, we can insert that value into Eqs. (9) and (11) to obtain optimal SQA's that are experimentally useful. In the leading-order parton processes for a dilepton with large Q_T, the large transverse momentum is balanced by that of a single parton. This recoiling parton produces a jet of hadrons whose momenta are nearly collinear to that of the parton. The polar angle of the recoiling parton in the hadron c.m. frame is approximately equal to the polar angle θ_{jet} of the jet. The ratio x_1/x_2 can be expressed as a function of θ_{jet} and the transverse and longitudinal momenta Q_T and Q_L of the dilepton in the hadron c.m. frame:

$$\left. \frac{x_1}{x_2} \right|_{\text{optimal}} = \frac{(E_{\gamma^*} + Q_L)\sin\theta_{\text{jet}} + Q_T(1 + \cos\theta_{\text{jet}})}{(E_{\gamma^*} - Q_L)\sin\theta_{\text{jet}} + Q_T(1 - \cos\theta_{\text{jet}})}, \quad (12)$$

where $E_{\gamma^*} = (Q_T^2 + Q_L^2 + Q^2)^{1/2}$. Inserting this into Eqs. (9) and (11), we obtain expressions for a/b for the optimal $q\bar{q}$ and qg axes that depend on quantities that can be directly measured.

Beyond leading order in α_s, there can be more than one parton in the final state, so there can be more than one jet with large transverse momentum. As a general prescription for x_1/x_2, we choose θ_{jet} in Eq. (12) to be the angle in the hadron c.m. frame of the jet with the largest transverse energy. Since the direction of the jet is insensitive to soft gluon radiation and to the splitting of a parton into collinear partons, QCD radiative corrections to the polar angle distributions defined by our optimal SQA's can be calculated systematically using perturbative QCD. The number of jets in an event depends on the size of the angular cone used to define the jet. If a reasonably large value is chosen for the cone size, most of the events will contain a single jet with large transverse

momentum balancing that of the dilepton. A small fraction of the events will have more than one such jet. For these multijet events, the expression on the right side of Eq. (12) may not be a good estimate for x_1/x_2. However since these events are only a small fraction of the total number of events, they should not seriously decrease the transverse polarization. Our expression for x_1/x_2 is not useful for fixed-target experiments, because the recoiling jet is usually not observed. However θ_{jet} can be measured relatively easily in high energy hadron colliders.

To illustrate the use of our optimal SQA's, we apply them to the production of a dilepton with invariant mass $Q = 3$ GeV at the Tevatron, which is a $p\bar{p}$ collider with c.m. energy 1.96 TeV. We calculate the cross sections for a longitudinal virtual photon using the CTEQ6L parton distributions [6] with 3 flavors (u, d, and s). The factorization and renormalization scales are set to $\mu = (Q^2 + Q_T^2)^{1/2}$. For the strong coupling constant $\alpha_s(\mu)$, we use the next-to-leading order formula with 4 flavors of quarks and Λ_{QCD}=326 MeV. For the electromagnetic coupling constant α, we use 1/132.6. We impose a rapidity cut $|y| < 1$ on the dilepton momentum.

We compare our optimal SQA's with three other choices for the SQA:

(i) the *c.m. helicity axis* defined by $X_{\text{cmh}} = P_1 + P_2$. The projection of the spin of the virtual photon along this axis is minus its helicity in the hadron c.m. frame.

(ii) the *Collins-Soper axis* defined by Eq. (7).

(iii) the *perpendicular helicity axis* defined by

$$X_{\perp \text{h}}^\mu = \frac{P_1^\mu}{Q \cdot P_1} + \frac{P_2^\mu}{Q \cdot P_2}. \quad (13)$$

In the dilepton rest frame, the perpendicuclar helicity axis is along $\boldsymbol{v}_1 + \boldsymbol{v}_2$. The spin of the virtual photon along this axis coincides with its helicity in the frame obtained from the hadron c.m. by a longitudinal boost that makes the dilepton momentum perpendicular to the beam direction. The perpendicular helicity axis coincides with the c.m. helicity axis when the rapidity of the dilepton is 0. One advantage shared by $X_{\perp \text{h}}$ and X_{CS} is that they are invariant under independent longitudinal boosts of the two colliding hadrons. Thus they can be expressed equally well in terms of the colliding parton momenta or the colliding hadron momenta.

A convenient polarization variable for a virtual photon is $\alpha = (\sigma_T - 2\sigma_L)/(\sigma_T + 2\sigma_L)$. It can be measured from the polar angle distribution of the lepton, which is proportional to $1 + \alpha \cos^2\theta$. The leading-order predictions for α for various choices of SQA are shown as functions of Q_T/Q in Fig. 1. For the Collins-Soper axis, α is negative for $Q_T > Q$. For the c.m. helicity axis, the perpendicular helicity axis, and our two optimal axes, α approaches 1 at large Q_T, indicating pure transverse polarization. The approach to 1 is significantly faster for the perpendicular helicity axis than for the c.m. helicity axis. However it
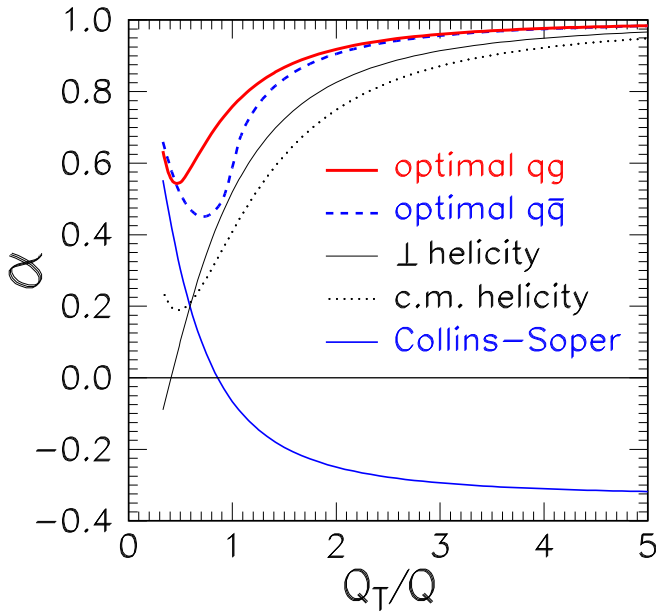
FIG. 1: (color online). Polarization variable α for various spin-quantization axes as functions of Q_T/Q for dileptons with invariant mass $Q = 3$ GeV at the Tevatron.

is even faster for both of the optimal axes. The two optimal axes give essentially the same α for $Q_T > 2Q$, but for $Q_T < 2Q$ α is closer to 1 for the optimal qg axis than for the optimal $q\bar{q}$ axis.

The requirement that the recoiling jet be captured by the hadronic calorimeter imposes a constraint on its pseudorapidity $\eta_{\rm jet} = \ln \tan(\theta_{\rm jet}/2)$. To provide some idea of how much this additional constraint might decrease the data sample, we impose the pseudorapidity cut $|\eta_{\rm jet}| < 1$. For dileptons with $Q = 3$ GeV at the Tevatron, the fraction of events satisfying the dilepton rapidity cut that also survive the jet pseudorapidity cut is greater than 0.5 for $Q_T > Q$. This fraction is large enough that measuring the angle $\theta_{\rm jet}$ should not dramatically decrease the size of the data sample.

We have also compared the various quantization axes for dilepton production at the LHC, which is a pp collider with c.m. energy 14 TeV. We imposed a rapidity cut $|y| < 3$ on the dilepton. For $Q_T > Q$, the leading-order results for α differ by less than 0.01 from the results for the Tevatron in Fig. 1 for all the SQA's with one exception. For the c.m. helicity axis, α at the LHC is smaller by more than 0.1 in the region near $Q_T = 1.5\ Q$. The leading-order predictions for α as functions of Q_T/Q are similar to those for the Tevatron in Fig. 1. If we impose a pseudorapidity cut $|\eta_{\rm jet}| < 3$ on the recoiling jet, the fraction of events satisfying the dilepton rapidity cut that also survive the jet pseudorapidity cut is greater than 0.8 for $Q_T > Q$.

The CDF and D0 Collaborations have measured α as a function of Q_T for dimuons produced at the Tevatron from decays of charmonium mesons [7] and bottomonium mesons [8, 9]. The variable α for the c.m. helicity axis

was measured for Q_T/Q as high as 9.7 for J/ψ and 2.1 for $\Upsilon(1S)$. The data samples collected by CDF and D0 are large enough that it should also be possible to measure α for the sidebands of these quarkonium resonances, which come from dileptons produced by virtual photons. At the LHC, it should be possible to measure α out to much larger values of Q_T/Q.

Our leading-order results for α as a function of Q_T indicate that the virtual photon will be significantly more strongly polarized at large transverse momentum if we use optimal SQA's. A quantitative prediction of the polarization requires calculations to next-to-leading order (NLO) in α_s. The angular distributions for lepton pairs at large Q_T have been calculated to NLO by Mirkes and Ohnemus [10]. At NLO, there can be two partons in the final state. In Ref. [10], the momenta of the two partons were integrated over, so they cannot be resolved into two separate jets. In order to use these NLO results, we would have to modify our prescription for x_1/x_2. At small Q_T, it is also necessary to sum up the effects of soft-gluon emission to all orders [4].

Similar methods could be used to derive optimal SQA's for the production at large transverse momentum of massive particles, such as J/ψ, Υ, and the weak bosons $W^{\pm}$ and Z^0. With an optimal SQA, these states should be more strongly polarized, so measurements of α will provide more information. It may also make the theoretical predictions more robust with respect to radiative corrections. Optimal SQA's might be useful for determining the spin of new particles created at the LHC. They provide a new window into the spins of particles created by parton collisions.

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